Quantum teleportation of a single-photon wave packet

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Abstract

A quantum teleportation scheme based on the EPR-pair entangled with respect to the "energy+time" variables is proposed. Teleportation of the multimode state of a single-photon wave packet is considered.

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The fundamental unit of quantum information is represented by a quantum bit (qubit) [1]. Qubit can be associated with arbitrary two-level quantum system (e.g. spin-1/2 particle). Any two orthogonal states of the quantum system can be identified with the boolean values 0 and 1 (states of a classical bit). Contrary to its classical counterpart (bit), the quantum beat can be found in an arbitrary superposition of the states 0 and 1. Different quantum bits can also be found in various entangled states. A quantum bit whose state is not known beforehand cannot be cloned [2]. There exists a fundamental law which prohibits cloning of an unknown quantum state [2,3]. Also, no single measurement can provide comprehensive information on the state of a quantum bit. However, a qubit can be reliably relayed by making its replica (quantum teleportation [4]). A fundamental difference between the clone and the copy is that the operator who has produced a replica of an unknown quantum bit does not know himself the state of the replica which is completely identical to the original quantum bit [4]. The quantum teleportation employs non-local quantum correlations (Einstein-Podolsky-Rosen effect) [5].

To produce a replica of an unknown state of the two-level quantum system (for example, spin-1/2 particle), user A first generates an EPR-pair, i.e. two spin-1/2 particles in an entangled state. Then he leaves one of the twins in his own apparatus and sends the other one to user B. After that user A performs joint measurements of the particle whose state is unknown to him and his twin from the EPR-pair. The measurements are made in the so-called Bell basis [6]. The measurements produce four equiprobable outcomes which can be reliably distinguished from each other because of the basis orthogonality (after the measurement the state of the pair is known). Because of the initial correlations in the EPR-pair, the particle sent to user B is rendered to a new state which is a perfect replica of the unknown state to within a unitary rotation [4]. The four outcomes of the measurement performed by user A yield two bits of classical information and indicate which unitary rotation should be applied by user B to his particle to produce the state identical to the initial unknown state. Reliable teleportation of a single qubit requires one EPR-pair (two qubits in an entangled state, i.e. one entangled bit (ebit [7])) and two classical bits of information.

Quantum teleportation has recently been demonstrated experimentally for a photon with unknown polarization [8].

The problem of teleportation of the wave function in the one-dimensional case where momentum and coordinate play the role of continuous dynamical variables was analyzed in Ref.[9] where the EPR-pair wave function was as a matter of fact replaced by a singular wave function from Ref. [5]:

$$\psi(x_1, x_2) \propto \delta(x_1 + x_2 - X_0)\delta(p_1 - p_2), \tag{1}$$

where (x_1, x_2) ? (p_1, p_2) correspond to canonically conjugate variables of the first and second particles in the EPR-pair. Such a singular wavefunction corresponds the ideal EPR correlations, and the closer the wavefunction to Eq.(1), the lower is the probability of obtaining the classical information sent by user A to user B, although the replica itself in the limit (1) tends to the ideal copy of the initial unknown state. Investigated in a recent paper [10] was the teleportation of a quantum state described by the dynamical variables (x, p) (the unknown states in Ref.[10] correspond to a single-mode of the photon) for the case of non-ideal EPR-correlations. Imperfect EPR correlations reduce the replica accuracy but simultaneously enhance the teleportation efficiency

In the present paper we propose a scheme for the teleportation of a single-photon wave packet employing the EPR pair which is in the entangled state with respect to the "energy+time" coordinates. EPR pairs of that kind are produced in the parametric down-conversion processes [11]. A fundamental difference between our case and the case considered in Ref.[10] is that the single-photon wave packet state is the multi-mode one, and the parameter of time appears in the problem explicitly.

To simplify the formulas, we shall assume that wave packet polarization is known. The arguments below can easily be extended to the case of unknown polarization by simply adding an extra subscript. The state of a single-photon wave packet can be written as [14]

$$|1\rangle_3 = \int_0^\infty d\omega f(\omega) e^{-i\omega t_0} \hat{a}^+(\omega)|0\rangle = \int_0^\infty d\omega f(\omega) e^{-i\omega t_0} |\omega\rangle_3,$$

$$[\hat{a}(\omega), \hat{a}^+(\omega')] = I\delta(\omega - \omega'), \quad \int_0^\infty |f(\omega)|^2 d\omega = 1,$$

where $\hat{a}^+(\omega)$, $\hat{a}(\omega)$ are the creation and annihilation operators of a single-mode Fock state $|\omega\rangle_3$, $|0\rangle$ is the vacuum state, $f(\omega)$ is the packet amplitude, t_0 is the initial moment of time which in the following will be assumed to be incorporated into the definition of $f(\omega)$. The density matrix at an arbitrary time is

$$\rho(3) = \left(\int_0^\infty d\omega e^{-i\omega t} f(\omega) |\omega\rangle_3 \right) \left(\int_0^\infty d\omega' {}_3\langle\omega| e^{-i\omega' t} f^*(\omega') \right) \tag{2}$$

The state of the EPR-pair of photons can be written as

$$|1_1, 1_2\rangle = \int_0^\infty \int_0^\infty d\omega d\omega' F(\omega, \omega') \hat{a}_1^+(\omega) \hat{a}_2^+(\omega')|0\rangle = \int_0^\infty \int_0^\infty d\omega d\omega' F(\omega, \omega') |\omega\rangle_1 \otimes |\omega'\rangle_2, \tag{3}$$

$$\rho_{EPR}(1, 2) = |1_1, 1_2\rangle\langle 1_1, 1_2|, \quad \int_0^\infty \int_0^\infty |F(\omega, \omega')|^2 d\omega d\omega' = 1,$$

where $F(\omega, \omega')$ is the joint amplitude of the photons in the EPR-pair. It is important that the amplitude $F(\omega, \omega')$ does not factorize: $(F(\omega, \omega') \neq f(\omega) \cdot f(\omega'))$.

According to the general scheme [12,13], quantum mechanical measurements are described by positive operators realizing the identity resolution. Corresponding to the observables associated with the self-adjoint operators are the orthogonal identity resolutions. Parameters (time, rotation angles) are not related to any self-adjoint operators, and therefore they correspond to non-orthogonal identity resolutions [12,13].

Let us first briefly discuss the joint measurements of the photons comprising an EPR-pair. Measurement of the energy (frequency) of one of the photons is described by the orthogonal identity resolution

$$\int_0^\infty E(d\Omega) = I, \quad E(d\Omega) = |\Omega\rangle\langle\Omega| d\Omega. \tag{4}$$

Accordingly, the measurement of time is given by the non-orthogonal identity resolution [12,13]

$$\int_{-\infty}^{\infty} M(dt) = I, \tag{5}$$

$$M(dt) = \left(\int_0^{\infty} e^{i\Omega t}|\Omega\rangle d\Omega\right)\left(\int_0^{\infty} e^{-i\Omega' t}\langle\Omega'|d\Omega'\right)\frac{dt}{2\pi}.$$

Joint measurement of the energy of photons in the EPR-pair yields the outcome probability distribution

$$\Pr\{d\omega_1 d\omega_2\} = \mathrm{Tr}\{\rho_{_{EPR}}(1,2)E(d\omega_1)E(d\omega_2)\} = |F(\omega_1,\omega_2)|^2 d\omega_1 d\omega_2, \tag{6}$$

where $E(d\omega_1)$, $E(d\omega_2)$ are the projectors on the single-mode Fock states of the first and second photon in the EPR-pair. Joint measurement of time (positive outcomes in the intervals (t_1, t_1+dt_1) and (t_2, t_2+dt_2)) generates the probability distribution

$$\Pr\{dt_1 dt_2\} = \mathrm{Tr}\{\rho_{_{EPR}}(1,2)M(dt_1)M(dt_2)\} = |F(t_1,t_2)|^2 dt_1 dt_2, \tag{7}$$

$$F(t_1,t_2) = \frac{1}{2\pi}\int_0^{\infty}\int_0^{\infty} F(\omega,\omega')e^{-i(\omega t_1+\omega' t_2)}d\omega d\omega'.$$

To illustrate the above arguments and simplify further analysis, we shall present the joint amplitude of two correlated variables in the form (e.g. see Ref.[15])

$$F(\omega,\omega') = \frac{1}{\sqrt{2\pi\sigma^2\sqrt{1-\mu^2}}}e^{-\frac{P(\omega,\omega')}{2}}, \tag{8}$$

where

$$P(\omega,\omega') = \frac{(\omega-\Omega_1)^2+(\omega'-\Omega_2)^2-2\mu(\omega-\Omega_1)(\omega'-\Omega_2)}{2\sigma^2(1-\mu^2)},$$

$$\Omega_0 = \Omega_1+\Omega_2,$$

where μ is the correlation coefficient. The frequency Ω_0 is assumed to be a known external parameter which is governed by the pumping frequency in the parametric down-conversion process.

Ideal EPR correlations (anticorrelation) correspond to $\mu = -1$ ($\mu = 1$). For $\mu \to -1$, $\sigma \to \infty$, and $\sigma^2(1-\mu^2) \to 0$, the joint measurement of the frequencies of the photons belonging to the EPR pair yields the following measurement outcomes probability distribution:

$$\Pr\{d\omega_1 d\omega_2\} \propto e^{-\frac{(\omega_1+\omega_2-\Omega_0)^2}{2\sigma^2(1-\mu^2)}} d\omega_1 d\omega_2 \to \delta(\omega_1+\omega_2-\Omega_0)d\omega_1 d\omega_2, \tag{9}$$

and, accordingly, measurement of time

$$\Pr\{dt_1 dt_2\} \propto e^{-\frac{\sigma^2(t_1+t_2+2\mu t_1 t_2)^2}{2}} dt_1 dt_2 \to \delta(t_1-t_2)dt_1 dt_2, \tag{10}$$

If the measurements are performed at spatially separated points, the times in Eq.(10) should be understood as the reduced times corrected for the photon times-of-flight, $(t_{1,2} \to t_{1,2} - \frac{x_{1,2}}{c})$. For brevity, in the rest of the paper we shall always use the reduced times without explicitly mentioning it.

Eqs. (9) and (10) represent the EPR effect for two complementary alternatives, the dynamical energy variable and the time parameter, similar to the EPR effect for the dynamical

variables of momentum and position [5]. If two distant users both perform the frequency measurements and one of them obtained a non-zero result at a frequency ω_1, the outcome of the second measurement can be predicted with certainty without actually doing it: non-zero outcome will occur for the frequency $\omega_2 = \Omega_0 - \omega_1$. However, if the time measurements were performed and the first user had a non-zero outcome at the time moment t_1, the moment of photon detection by the second user is bound to be $t_2 = t_1 - \frac{x_2 - x_1}{c}$.

The idea of teleportation applied to the present case is to employ a joint (entangled) "energy+time" measurement of the pair of photons one of which belongs to the EPR-pair and the second one is in an unknown state. The indicated measurement is given by the non-orthogonal identity resolution

$$\int\int M(dtd\Omega_-) = I, \tag{11}$$

$$M(dtd\Omega_-) =$$

$$\left(\int d\Omega_+ e^{i\Omega_+ t}|\Omega_+ + \Omega_-\rangle_1 \otimes |\Omega_+ - \Omega_-\rangle_3\right)\left(\int d\Omega'_+ e^{-i\Omega'_+ t}\,{}_3\langle\Omega'_+ - \Omega_-| \otimes {}_1\langle\Omega'_+ + \Omega_-|\right)\frac{dtd\Omega_-}{2\pi} \tag{12}$$

Here integration is performed over the frequencies resulting in positive arguments of the Fock states. It should be emphasized that the frequency Ω_- is common to all bra and ket states. Indeed, the integration over t in Eq.(11) yields $\delta(\Omega_+ - \Omega'_+)$, and further integration over Ω'_+ eliminates one integral over Ω'_+ resulting in

$$\int\int d\Omega_- d\Omega_+ \left(|\Omega_+ + \Omega_-\rangle_1 \otimes |\Omega_+ - \Omega_-\rangle_3\right)\left({}_3\langle\Omega_+ - \Omega_-| \otimes {}_1\langle\Omega'_+ + \Omega_-|\right) = \tag{13}$$

$$\int_0^\infty\int_0^\infty d\omega_1 d\omega_3 |\omega_1\rangle_1 \otimes |\omega_3\rangle_3\ {}_3\langle\omega_3| \otimes {}_1\langle\omega_1| = I_1 \otimes I_3,$$

where $\omega_1 = \Omega_+ + \Omega_-$? $\omega_3 = \Omega_+ - \Omega_-$. In some sense the measurement (11) resembles a partial Fourier transform over the sum frequency of the two photons (while Fourier transform over the difference frequency is not performed). The probability to find the measured quantities within the intervals $(t, t+dt)$? $(\Omega_-, \Omega_- + d\Omega_-)$ is described by the formula

$$\Pr\{dtd\Omega_-\} = \left(\int d\omega_2 \int\int d\Omega_+ d\Omega'_+ \tilde{F}(\Omega_+ + \Omega_- + \omega_2; \Omega_+ + \Omega_- - \omega_2)\right. \tag{14}$$

$$\left.\tilde{F}^*(\Omega'_+ + \Omega_- + \omega_2; \Omega'_+ + \Omega_- - \omega_2) f(\Omega_+ - \Omega_-) f^*(\Omega'_+ - \Omega_-) e^{i(\Omega_+ - \Omega'_+)t}\right)\frac{dtd\Omega_-}{2\pi}$$

Here we have introduced for convenience the notation $\tilde{F}(\omega + \omega'; \omega - \omega') \equiv F(\omega; \omega')$. After the measurement user A sends the obtained values of t and Ω_- through a public channel to user B who uses them to reconstruct the state to be teleported.

In the limit of ideal EPR-correlations: $\sigma^2 \to \infty$ and $\sigma^2(1 - \mu^2) \to 0$ $(\mu \to -1)$,

$$\tilde{F}(\omega + \omega'; \omega - \omega') \propto \delta(\omega + \omega' - \Omega_0) \cdot const(\omega - \omega'), \tag{15}$$

where $const(\omega - \omega')$ is a function which is almost constant in a wide range of its argument. After the measurement performed by user A, the state of the second photon in the EPR-pair observed by user B is given by the density matrix

$$\rho(2) = \frac{\mathrm{Tr}\{\rho_{EPR}(1,2) \otimes \rho(3) M(dtd\Omega_-)\}}{\Pr\{dtd\Omega_-\}} \tag{16}$$

In the limit of ideal EPR-correlations the state (15) observed by user B, taking into account Eq.(14), tends to

$$\rho(2) \to \left(\int_0^\infty d\omega_2 e^{-i\omega_2 t} f(\omega_0 - \omega_2) |\omega_2\rangle_2 \right) \left(\int_0^\infty d\omega_2' {}_2\langle \omega_2' | e^{i\omega_2' t} f^*(\omega_0 - \omega_2') \right), \tag{17}$$

where $\omega_0 = \Omega_0 - \Omega_-$.

The density matrix (17) is almost identical to the original density matrix to within a frequency shift in the argument $f(\omega_0 - \omega_2)$. The measurement similar to Eq.(12) can be used when the continuous dynamical variables we are dealing with are the momentum and position. In that case it can be written in the form

$$\int\int M(dp_- dx_+) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{dp_- dx_+}{2\pi} \left(\int_{-\infty}^{\infty} dx_- e^{ix_- p_-} |x_- + x_+\rangle_1 \otimes |x_- - x_+\rangle_3 \right)$$
$$\left(\int_{-\infty}^{\infty} dx'_- e^{-ix'_- p_-} {}_3\langle x'_- - x_+| \otimes {}_1\langle x'_- + x_+| \right), \tag{18}$$

??? $x_\pm = x_1 \pm x_2$, $x'_\pm = x'_1 \pm x'_2$, $p_+ = p_1 + p_2$. A fundamental difference between Eqs (12) and (18) is that the position and momentum shifts comprise a group. On the contrary, in the case of energy and time the frequency shifts comprise a semigroup since frequency belongs to a semi-infinite interval $(0, \infty)$ [12,13]. Therefore, in the "position+momentum" teleportation scheme the shift in the state amplitude argument $f(x_0 - x_2)$ (see Eq.(17)) is of no importance and can be eliminated by choosing an appropriate reference frame (shifting the argument $|x_2\rangle \to |x_0 - x_2\rangle$; this shift is allowed since the argument changes within the infinite interval $(-\infty, \infty)$). In our opinion, in our case the arising frequency shift cannot be eliminated by a similar modification of the argument since frequency can only take positive values. In this way the peculiarity of time which is a parameter rather than a dynamical variable in quantum mechanics is manifested.

In the limit when the joint amplitude in the EPR-pair $\tilde{F}$ as a delta-function as a function of sum frequency and a constant as a function of difference frequency, Eq.14 implies that the outcome probability distribution does not depend on time. In that case the probability to obtain a non-zero result is the same throughout the whole interval of t $(-\infty, \infty)$. The latter means that the ideal teleportation is necessarily associated with the process efficiency tending to zero, since user A receives the classical information with only vanishingly small probability. When a continuous variable is teleported, the replica cannot be a perfect copy of the unknown state.

The problem of teleportation optimization when dealing with the continuous parameters or dynamical variables still remains open. The measurement (12) can be realized by mixing two photons (the unknown photon and one of the photons belonging to the EPR-pair) by a beam splitter and subsequent measurements by narrow-band (for Ω_-) and wide-band (for t) photodetectors in the two channels.

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